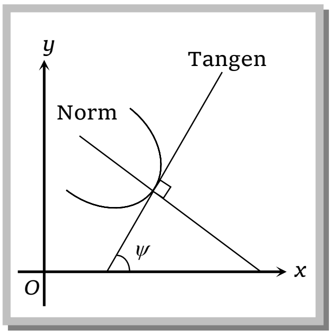
4.2 Tangent and Normal

4.2.1 Slope of the Tangent and Normal

(1) **Slope of the tangent :** If tangent is drawn on the curve $y = f(x)$ at point $P(x_1, y_1)$ and this tangent makes an angle ψ with positive x-direction then,



$$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \tan\psi = \text{slope of the tangent}$$

Note : ❑ If tangent is parallel to x-axis $\psi = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 0$

❑ If tangent is perpendicular to x-axis $\psi = \frac{\pi}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \infty$

(2) **Slope of the normal :** The normal to a curve at $P(x_1, y_1)$ is a line perpendicular to the tangent at P and passing through P and slope of the normal = $\frac{-1}{\text{Slope of tangent}}$ = $\frac{-1}{\left(\frac{dy}{dx}\right)_{P(x_1,y_1)}} = -\left(\frac{dx}{dy}\right)_{P(x_1,y_1)}$

Note : ❑ If normal is parallel to x-axis

$$\Rightarrow -\left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0 \text{ or } \left(\frac{dx}{dy}\right)_{(x_1,y_1)} = 0$$

❑ If normal is perpendicular to x-axis (for parallel to y-axis)

$$\Rightarrow -\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 0$$

Example: 1 The slope of the tangent to the curve $x^2 + y^2 = 2c^2$ at point (c, c) is **[AMU 1998]**

(a) 1 (b) – 1 (c) 0 (d) 2

Solution: (b) Given $x^2 + y^2 = 2c^2$

Differentiating *w.r.t. x*, $2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow 2y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = \frac{-x}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(c,c)} = -1$

Example: 2 The line $x + y = 2$ is tangent to the curve $x^2 = 3 - 2y$ at its point **[MP PET 1998]**

(a) (1, 1) (b) (−1, 1) (c) $(\sqrt{3}, 0)$ (d) (3, – 3)

Solution: (a) Given curve $x^2 = 3 - 2y$

diff. *w.r.t. x*, $2x = -\frac{2dy}{dx}$; $\frac{dy}{dx} = -x$

Slope of the line = – 1

$\frac{dy}{dx} = -x = -1$; $x = 1$

$\therefore y = 1$ point (1, 1)

Example: 3 The tangent to the curve $y = 2x^2 - x + 1$ at a point P is parallel to $y = 3x + 4$, the co-ordinate of P are **[Rajasthan P**

(a) (2, 1) (b) (1, 2) (c) (– 1, 2) (d) (2, – 1)

Solution: (b) Given $y = 2x^2 - x + 1$

Let the co-ordinate of P is (h, k) then $\left(\frac{dy}{dx}\right)_{(h,k)} = 4h - 1$

Clearly $4h - 1 = 3$

$h = 1 \Rightarrow k = 2$. P is (1, 2).

4.2.2 Equation of the Tangent and Normal

(1) **Equation of the tangent :** We know that the equation of a line passing through a point $P(x_1, y_1)$ and having slope m is $y - y_1 = m(x - x_1)$

Slope of the tangent at (x_1, y_1) is = $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$

The equation of the tangent to the curve $y = f(x)$ at point $P(x_1, y_1)$ is

$$\boxed{y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)}$$

(2) **Equation of the normal :** Slope of the Normal = $\frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}$

Thus equation of the normal to the curve $y = f(x)$ at point $P(x_1, y_1)$

$$\boxed{y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x - x_1)}$$

Note : ❑ If at any point $P(x_1, y_1)$ on the curve $y = f(x)$, the tangent makes equal angle with the axes, then at the point P, $\psi = \frac{\pi}{4}$ or $\frac{3\pi}{4}$. Hence, at P $\tan\psi = \frac{dy}{dx} = \pm 1$.

Example: 4 The equation of the tangent at $(-4, -4)$ on the curve $x^2 = -4y$ is **[Karnataka CET 2001]**

(a) $2x + y + 4 = 0$ (b) $2x - y - 12 = 0$ (c) $2x + y - 4 = 0$ (d) $2x - y + 4 = 0$

Solution: (d) $x^2 = -4y \Rightarrow 2x = -4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-x}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{(-4,-4)} = 2$.

We know that equation of tangent is $(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1) \Rightarrow y + 4 = 2(x + 4) \Rightarrow 2x - y + 4 = 0$.

Example: 5 The equation of the normal to the curve $y = \sin\frac{\pi x}{2}$ at (1, 1) is

(a) $y = 1$ (b) $x = 1$ (c) $y = x$ (d) $y - 1 = \frac{-2}{\pi}(x - 1)$

Solution: (b) $y = \sin\frac{\pi x}{2} \Rightarrow \frac{dy}{dx} = \frac{\pi}{2}\cos\frac{\pi}{2}x \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 0$

$\therefore$ Equation of normal is $y - 1 = \frac{1}{0}(x - 1) \Rightarrow x = 1$.

Example: 6 The equation of the tangent to the curve $y = be^{-x/a}$ at the point where it crosses y-axis is

(a) $ax + by = 1$ (b) $ax - by = 1$ (c) $\frac{x}{a} - \frac{y}{b} = 1$ (d) $\frac{x}{a} + \frac{y}{b} = 1$

Solution: (d) Curve is $y = be^{-x/a}$

Since the curve crosses y-axis (*i.e.*, $x = 0$) $\therefore$ $y = b$

Now $\frac{dy}{dx} = \frac{-b}{a}e^{-x/a}$. At point (0, b), $\left(\frac{dy}{dx}\right)_{(0,b)} = \frac{-b}{a}$

$\therefore$ Equation of tangent is $y - b = \frac{-b}{a}(x - 0) \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$.

Example: 7 If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x-axis then $f'(3)$ is equal to

[IIT Screening 2000; DCE 2001]

(a) – 1 (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) 1

Solution: (d) Slope of the normal $= \frac{-1}{dy/dx} \Rightarrow \tan\frac{3\pi}{4} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(3,4)}}$

$\therefore \left(\frac{dy}{dx}\right)_{(3,4)} = 1$; $f'(3) = 1$.

Example: 8 The point (s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical (parallel to y-axis), is are**[IIT Screenin**

(a) $\left[\pm\frac{4}{\sqrt{3}}, -2\right]$ (b) $\left(\pm\frac{\sqrt{11}}{3}, 1\right)$ (c) $(0, 0)$ (d) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

Solution: (d) $y^3 + 3x^2 = 12y$

$\Rightarrow 3y^2.\frac{dy}{dx}+6x=12.\frac{dy}{dx} \Rightarrow \frac{dy}{dx}(3y^2-12)+6x=0 \Rightarrow \frac{dy}{dx}=\frac{6x}{12-3y^2} \Rightarrow \frac{dx}{dy}=\frac{12-3y^2}{6x}$

Tangent is parallel to y-axis, $\frac{dx}{dy}=0 \Rightarrow 12-3y^2$ = 0 or $y=\pm 2$. Then $x=\pm\frac{4}{\sqrt{3}}$, for $y=2$

$y=-2$ does not satisfy the equation of the curve, $\therefore$ The point is $\left(\pm\frac{4}{\sqrt{3}},2\right)$

Example: 9 At which point the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-x/a}$ **[Rajasthan PET 1999]**

(a) (0, 0) (b) (0, a) (c) (0, b) (d) (b, 0)

Solution: (c) Let the point be (x_1,y_1) $\therefore y_1=be^{-x_1/a}$ ......(i)

Also, curve $y=be^{-x/a} \Rightarrow \frac{dy}{dx}=\frac{-b}{a}e^{-x/a}$

$\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=\frac{-b}{a}e^{-x_1/a}=\frac{-y_1}{a}$ (by (i))

Now, the equation of tangent of given curve at point (x_1,y_1) is $y-y_1=\frac{-y_1}{a}(x-x_1) \Rightarrow \frac{x}{a}+\frac{y}{y_1}=\frac{x_1}{a}+1$

Comparing with $\frac{x}{a}+\frac{y}{b}=1$, we get, $y_1=b$ and $1+\frac{x_1}{a}=1 \Rightarrow x_1=0$

Hence, the point is (0, b).

Example: 10 The abscissa of the point, where the tangent to curve $y=x^3-3x^2-9x+5$ is parallel to x-axis are **[Karnataka C**

(a) 0 and 0 (b) $x=1$ and -1 (c) $x=1$ and -3 (d) $x=-1$ and 3

Solution: (d) $y=x^3-3x^2-9x+5 \Rightarrow \frac{dy}{dx}=3x^2-6x-9$.

We know that this equation gives the slope of the tangent to the curve. The tangent is parallel to x-axis $\frac{dy}{dx}=0$

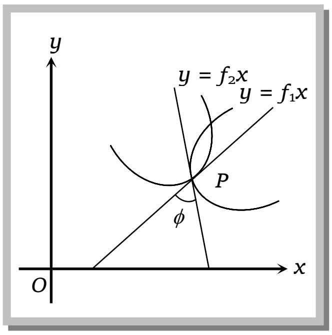
Therefore, $3x^2-6x-9=0 \Rightarrow x=-1,3$.

4.2.3 Angle of Intersection of Two Curves

The angle of intersection of two curves is defined to be the angle between the tangents to the two curves at their point of intersection.

We know that the angle between two straight lines having slopes

$$\phi=\tan^{-1}\frac{m_1-m_2}{1+m_1m_2}$$



Also slope of the tangent at $P(x_1,y_1)$

$$m_1=\left(\frac{dy}{dx}\right)_{1(x_1,y_1)}, \quad m_2=\left(\frac{dy}{dx}\right)_{2(x_1,y_1)}$$

Thus the angle between the tangents of the two curves $y=f_1(x)$ and $y=f_2(x)$

$$\boxed{\tan\phi = \frac{\left(\frac{dy}{dx}\right)_{1(x_1,y_1)} - \left(\frac{dy}{dx}\right)_{2(x_1,y_1)}}{1+\left(\frac{dy}{dx}\right)_{1(x_1,y_1)}\left(\frac{dy}{dx}\right)_{2(x_1,y_1)}}}$$

Orthogonal curves : If the angle of intersection of two curves is right angle, the two curves are said to intersect orthogonally. The curves are called orthogonal curves. If the curves are orthogonal, then $\phi = \frac{\pi}{2}$

$$m_1 m_2 = -1 \Rightarrow \left(\frac{dy}{dx}\right)_1 \left(\frac{dy}{dx}\right)_2 = -1$$

Example: 11 The angle between the curves $y^2 = x$ and $x^2 = y$ at (1, 1) is

(a) $\tan^{-1}\frac{4}{3}$ (b) $\tan^{-1}\frac{3}{4}$ (c) 90^o (d) 45^o

Solution: (b) Given curve $y^2 = x$ and $x^2 = y$

Differentiating *w.r.t. x*, $2y\frac{dy}{dx} = 1$ and $2x = \frac{dy}{dx}$

$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2}$ and $\left(\frac{dy}{dx}\right)_{(1,1)} = 2$

Angle between the curve

$\Rightarrow \tan\phi = \frac{2-\frac{1}{2}}{1+\frac{1}{2}.2} \Rightarrow \tan\phi = \frac{3}{4} \Rightarrow \phi = \tan^{-1}\frac{3}{4}$.

Example: 12 If the two curves $y = a^x$ and $y = b^x$ intersect at α, then $\tan\alpha$ equal **[MP PET 2001]**

(a) $\frac{\log a - \log b}{1+\log a \log b}$ (b) $\frac{\log a + \log b}{1-\log a \log b}$ (c) $\frac{\log a - \log b}{1-\log a \log b}$ (d) None of these

Solution: (a) Clearly the point of intersection of curves is (0, 1)

Now, slope of tangent of first curve, $m_1 = \frac{dy}{dx} = a^x \log a \Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = m_1 = \log a$

Slope of tangent of second curve, $m_2 = \frac{dy}{dx} = b^x \log b \Rightarrow m_2 = \left(\frac{dy}{dx}\right)_{(0,1)} = \log b$

$\therefore \tan\alpha = \frac{m_1 - m_2}{1+m_1 m_2} = \frac{\log a - \log b}{1+\log a \log b}$.

Example: 13 The angle of intersection between curve $xy = 6$ and $x^2 y = 12$

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{3}{11}\right)$ (c) $\tan^{-1}\left(\frac{11}{3}\right)$ (d) 0^o

Solution: (b) The equation of two curves are $xy = 6$ and $x^2 y = 12$ from (i) we obtain $y = \frac{6}{x}$ putting this value of y in equation (ii) to obtain $x^2\left(\frac{6}{x}\right) = 12 \Rightarrow 6x = 12 \Rightarrow x = 2$

Putting $x = 2$ in (i) or (ii) we get, $y = 3$. Thus, the two curves intersect at P(2, 3)

Differentiating (i) *w.r.t. x*, we get $x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{x} \Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = -\frac{3}{2} = m_1$

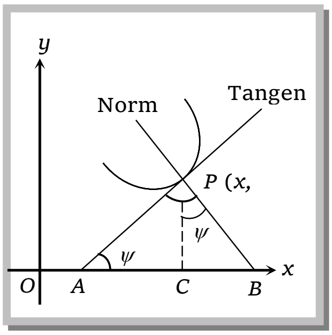
Differentiating (ii) *w.r.t. x*, we get $x^2\frac{dy}{dx}+2xy=0 \Rightarrow \frac{dy}{dx}=\frac{-2y}{x}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)}=-3=m_2 \Rightarrow \tan\theta=\frac{m_1-m_2}{1+m_1m_2}=\left(\frac{-3}{2}+3\right)\Big/\left(1+\left(\frac{-3}{2}\right)(-3)\right)=\frac{3}{11} \Rightarrow \theta=\tan^{-1}\frac{3}{11}$.

4.2.4 Length of Tangent, Normal, Subtangent and Subnormal

Let the tangent and normal at point $P(x,y)$ on the curve $y=f(x)$ meet the x-axis at points A and B respectively. Then PA and PB are called length of tangent and normal respectively at point P. If PC be the perpendicular from P on x-axis, the AC and BC are called length of subtangent and subnormal respectively at P. If PA makes angle ψ with x-axis, then $\tan\psi=\frac{dy}{dx}$ from fig., we find that

(1) Length of tangent $PA=y\operatorname{cosec}\psi=y\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\left(\frac{dy}{dx}\right)}$



(2) Length of normal $PB=y\sec\psi=y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

(3) Length of subtangent $AC=y\cot\psi=\frac{y}{\left(\frac{dy}{dx}\right)}$

(4) Length of subnormal $BC=y\tan\psi=y\left(\frac{dy}{dx}\right)$

Example: 14 The length of subtangent to the curve $x^2y^2=a^4$ at the point $(-a,a)$ is **[Karnataka CET 2001]**

(a) $3a$ (b) $2a$ (c) a (d) $4a$

Solution: (c) Equation of the curve $x^2y^2=a^4$.

Differentiating the given equation,

$$x^2 2y\frac{dy}{dx}+y^2 2x=0 \Rightarrow \frac{dy}{dx}=\frac{-y}{x} \Rightarrow \left(\frac{dy}{dx}\right)_{(-a,a)}=-\left(\frac{a}{-a}\right)=1$$

Therefore, sub-tangent = $\frac{y}{\left(\frac{dy}{dx}\right)}=a$.

Example: 15 For the curve $y^n=a^{n-1}x$, the sub-normal at any point is constant, the value of n must be

(a) 2 (b) 3 (c) 0 (d) 1

Solution: (a) $y^n=a^{n-1}x \Rightarrow ny^{n-1}\frac{dy}{dx}=a^{n-1} \Rightarrow \left(\frac{dy}{dx}\right)=\frac{a^{n-1}}{ny^{n-1}}$

$\therefore$ Length of the subnormal = $y\dfrac{dy}{dx}=\dfrac{ya^{n-1}}{ny^{n-1}}=\dfrac{a^{n-1}y^{2-n}}{n}$

We also know that if the subnormal is constant, then $\dfrac{a^{n-1}}{n}.y^{2-n}$ should not contain y.

Therefore, $2-n=0$ or $n=2$.

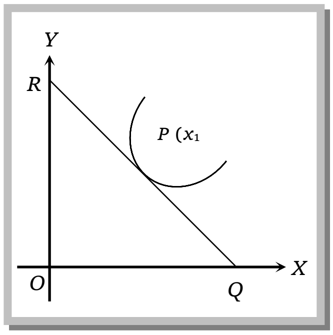
4.2.5 Length of Intercept made on Axis by the Tangent

Equation of tangent at any point (x_1,y_1) to the curve $y = f(x)$ is $y-y_1=\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}(x-x_1)$ ......(i)

Equation of x-axis y = 0 ......(ii)

and Equation of y-axis x = 0 ......(iii)

Solving (i) and (ii) we get $x=x_1-\left[\dfrac{y_1}{\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}}\right]$



x-intercept $OQ=x_1-\left[\dfrac{y_1}{\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}}\right]$

Similarly solving (i) and (iii) we get, y-intercept OR = $y_1-\left[x_1\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}\right]$

Example: 16 The sum of intercepts on co-ordinate axes made by tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is

(a) a (b) 2a (c) $2\sqrt{a}$ (d) None of these

Solution: (a) $\sqrt{x}+\sqrt{y}=\sqrt{a} \Rightarrow \dfrac{1}{2\sqrt{x}}+\dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx}=0 \Rightarrow \therefore \dfrac{dy}{dx}=-\dfrac{\sqrt{y}}{\sqrt{x}}$

Hence tangent at (x, y) is $Y-y=-\dfrac{\sqrt{y}}{\sqrt{x}}(X-x)$ or $X\sqrt{y}+Y\sqrt{x}=\sqrt{xy}(\sqrt{x}+\sqrt{y})=\sqrt{axy}$ or $\dfrac{X}{\sqrt{a}\sqrt{x}}+\dfrac{Y}{\sqrt{a}\sqrt{y}}=1$.

Clearly its intercepts on the axes are $\sqrt{a}\sqrt{x}$ and $\sqrt{a}\sqrt{y}$.

Sum of the intercepts = $\sqrt{a}(\sqrt{x}+\sqrt{y})=\sqrt{a}.\sqrt{a}=a$.

4.2.6 Length of Perpendicular from Origin to the Tangent

Length of perpendicular from origin (0, 0) to the tangent drawn at point $P(x_1,y_1)$ of the curve $y = f(x)$

$$p=\left|\frac{y_1-x_1\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}}{\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}}\right|$$

Example: 17 The length of perpendicular from (0, 0) to the tangent drawn to the curve $y^2 = 4(x+2)$ at point (2, 4) is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{3}{\sqrt{5}}$ (c) $\frac{6}{\sqrt{5}}$ (d) 1

Solution: (c) Differentiating the given equation *w.r.t.* x, $2y\frac{dy}{dx} = 4$ at point (2, 4) $\frac{dy}{dx} = \frac{1}{2}$

$$P = \frac{y_1 - x_1\left(\frac{dy}{dx}\right)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \frac{4-2\left(\frac{1}{2}\right)}{\sqrt{1+\frac{1}{4}}} = \frac{6}{\sqrt{5}}.$$

Assignment

Tangent and Normal

Basic Level

1. If the line $y = 2x + k$ is a tangent to the curve $x^2 = 4y$, then k is equal to **[AMU 2002]**

(a) 4 (b) $\frac{1}{2}$ (c) – 4 (d) $-\frac{1}{2}$

2. The point on the curve $y^2 = x$ where tangent makes 45^o angle with x-axis is **[Rajasthan PET 1990, 92]**

(a) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (c) (4, 2) (d) (1, 1)

3. If $x = t^2$ and $y = 2t$, then equation of the normal at $t = 1$ is

(a) $x + y - 3 = 0$ (b) $x + y - 1 = 0$ (c) $x + y + 1 = 0$ (d) $x + y + 3 = 0$

4. If normal to the curve $y = f(x)$ is parallel to x-axis, then correct statement is **[Rajasthan PET 2000]**

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 0$ (d) None of these

5. The equation of the tangent to the curve $(1 + x^2)y = 2 - x$, where it crosses the x-axis, is

(a) $x + 5y = 2$ (b) $x - 5y = 2$ (c) $5x - y = 2$ (d) $5x + y - 2 = 0$

6. The equation of tangent to the curve $y = 2\cos x$ at $x = \frac{\pi}{4}$ is

(a) $y - \sqrt{2} = 2\sqrt{2}\left(x - \frac{\pi}{4}\right)$ (b) $y + \sqrt{2} = \sqrt{2}\left(x + \frac{\pi}{4}\right)$ (c) $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$ (d) $y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

7. For the curve $x = t^2 - 1,\ y = t^2 - t$, the tangent line is perpendicular to x-axis where **[MNR 1980]**

(a) $t = 0$ (b) $t = \infty$ (c) $t = \frac{1}{\sqrt{3}}$ (d) $t = -\frac{1}{\sqrt{3}}$

8. If at any point on a curve the sub-tangent and subnormal are equal, then the tangent is equal to

(a) Ordinate (b) $\sqrt{2}$ ordinate (c) $\sqrt{2\,(\text{ordinate})}$ (d) None of these

9. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts, α and β on the coordinate axes such that $\alpha^2 + \beta^2 = 61$, then a =

(a) ± 30 (b) ± 5 (c) ± 6 (d) ± 61

10. If the tangent to the curve $x = a(\theta + \sin\theta), y = a(1 + \cos\theta)$ at $\theta = \frac{\pi}{3}$ makes an angle α with x-axis, then α =

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

11. If the tangent to the curve $xy + ax + by = 0$ at (1, 1) is inclined at an angle $\tan^{-1} 2$ with x-axis, then

(a) $a = 1, b = 2$ (b) $a = 1, b = -2$ (c) $a = -1, b = 2$ (d) $a = -1, b = -2$

12. The fixed point P on the curve $y = x^2 - 4x + 5$ such that the tangent at P is perpendicular to the line $x + 2y - 7 = 0$ is given by

(a) (3, 2) (b) (1, 2) (c) (2, 1) (d) None of these

13. The points of contact of the tangents drawn from the origin to the curve $y = \sin x$ lie on the curve

(a) $x^2 - y^2 = xy$ (b) $x^2 + y^2 = x^2y^2$ (c) $x^2 - y^2 = x^2y^2$ (d) None of these

14. The slope of the tangent to the curve $y^2 = 4ax$ drawn at point $(at^2, 2at)$ is **[Rajasthan PET 1993]**

(a) t (b) $\frac{1}{t}$ (c) $-t$ (d) $\frac{-1}{t}$

15. The slope of the curve $y = \sin x + \cos^2 x$ is zero at the point, where

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) No where

16. The equation of tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point (x_1, y_1) is

(a) $\frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \frac{1}{\sqrt{a}}$ (b) $\frac{x}{\sqrt{x_1}} + \frac{y}{\sqrt{y_1}} = \sqrt{a}$ (c) $x\sqrt{x_1} + y\sqrt{y_1} = \sqrt{a}$ (d) None of these

17. A tangent to the curve $y = x^2 + 3x$ passes through a point (0, – 9) if it is drawn at the point

(a) (– 3, 0) (b) (1, 4) (c) (0, 0) (d) (– 4, 4)

18. The sum of the intercepts made by a tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ at point (4, 4) on coordinate axes is

(a) $4\sqrt{2}$ (b) $6\sqrt{3}$ (c) $8\sqrt{2}$ (d) $\sqrt{256}$

19. The angle of intersection between the curve $y^2 = 16x$ and $2x^2 + y^2 = 4$ is **[Rajasthan PET 1993]**

(a) 0^o (b) 30^o (c) 45^o (d) 90^o

20. The equation of normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sec\theta, b\tan\theta)$ is

(a) $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$ (b) $\frac{ax}{\sec\theta} - \frac{by}{\tan\theta} = a^2 - b^2$ (c) $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 - b^2$ (d) $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a - b$

21. If tangent to a curve at a point is perpendicular to x-axis, then at the point

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dx}{dy} = 0$ (c) $\frac{dy}{dx} = 1$ (d) $\frac{dy}{dx} = -1$

22. If m be the slope of a tangent to the curve $e^y = 1 + x^2$ then

(a) $|m| > 1$ (b) $m < 1$ (c) $|m| < 1$ (d) $|m| \le 1$

23. The equation of the tangent to the curve $y = e^{-|x|}$ at the point where the curve cuts the line $x = 1$ is

(a) $x + y = e$ (b) $e(x + y) = 1$ (c) $y + ex = 1$ (d) None of these

24. The slope of the tangent to the curve $y = \int_0^x \frac{dx}{1+x^3}$ at the point where $x = 1$ is

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) None of these

25. The angle of intersection between the curves $x^2 = 4ay$ and $y^2 = 4ax$ at origin is **[Rajasthan PET 1997]**

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

26. The equation of the normal to the curve $y = x(2-x)$ at the point (2, 0) is **[Rajasthan PET 1989, 1992]**

(a) $x - 2y = 2$ (b) $x - 2y + 2 = 0$ (c) $2x + y = 4$ (d) $2x + y - 4 = 0$

27. The angle of intersection of the curve $y = 4 - x^2$ and $y = x^2$ is
[Rajasthan PET 1989, 1993; MNR 1978]

(a) $\frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$ (c) $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$ (d) None of these

28. Tangent to the curve $y = e^{2x}$ at point (0, 1) meets *x*-axis at the point

(a) (0, a) (b) (2, 0) (c) $\left(-\frac{1}{2}, 0\right)$ (d) Non where

29. The equation of the tangent to the curve $x = a\cos^3 t, y = a\sin^3 t$ at 't' point is **[Rajasthan PET 1988]**

(a) $x\sec t - y\,\text{cosec}\,t = a$ (b) $x\sec t + y\,\text{cosec}\,t = a$ (c) $x\,\text{cosec}\,t - y\sec t = a$ (d) $x\,\text{cosec}\,t + y\sec t = a$

30. The length of the tangent to the curve $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$ is

(a) ax (b) ay (c) a (d) xy

31. The point at the curve $y = 12x - x^3$ where the slope of the tangent is zero will be **[Rajasthan PET 1992]**

(a) (0, 0) (b) (2, 16) (c) (3, 9) (d) None of these

32. The angle of intersection between the curves $y = x^2$ and $4y = 7 - 3x^3$ at point (1, 1) is **[Andhra CEE 1992]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) None of these

Advance Level

33. Consider the following statements:

Assertion (*A*) : The circle $x^2 + y^2 = 1$ has exactly two tangents parallel to the *x*-axis

Reason (*R*) : $\frac{dy}{dx} = 0$ on the circle exactly at the points $(0, \pm 1)$. Of these statements **[SCRA 1996]**

(a) Both A and R and true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

34. The slope of the tangent to the curve $x = 3t^2 + 1, y = t^3 - 1$ at $x = 1$ is

(a) 0 (b) $\frac{1}{2}$ (c) ∞ (d) -2

35. The slope of tangent to the curve $x = t^2 + 3t - 8,\ y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is **[MNR 1994]**

(a) $\frac{22}{7}$ (b) $\frac{6}{7}$ (c) -6 (d) None of these

36. At what points of the curve $y = \frac{2}{3}x^3 + \frac{1}{2}x^2$, tangent makes the equal angle with axis **[UPSEAT 1999]**

(a) $\left(\frac{1}{2}, \frac{5}{24}\right)$ and $\left(-1, -\frac{1}{6}\right)$ (b) $\left(\frac{1}{2}, \frac{4}{9}\right)$ and $(-1, 0)$ (c) $\left(\frac{1}{3}, \frac{1}{47}\right)$ and $\left(-1, \frac{1}{3}\right)$ (d) $\left(\frac{1}{3}, \frac{1}{7}\right)$ and $\left(-3, \frac{1}{2}\right)$

37. For the curve $xy = c^2$ the subnormal at any point varies as

(a) x^2 (b) x^3 (c) y^2 (d) y^3

38. The point of the curve $y^2 = 2(x-3)$ at which the normal is parallel to the line $y - 2x + 1 = 0$ is

(a) (5, 2) (b) $\left(-\frac{1}{2}, -2\right)$ (c) (5, – 2) (d) $\left(\frac{3}{2}, 2\right)$

39. Coordinates of a point on the curve $y = x \log x$ at which the normal is parallel to the line $2x - 2y = 3$ are **[Rajasthan PET**

(a) (0, 0) (b) (e, e) (c) $(e^2, 2e^2)$ (d) $(e^{-2}, -2e^{-2})$

40. The abscissa of the points of curve $y = x(x-2)(x-4)$ where tangents are parallel to x-axis is obtained as

(a) $x = 2 \pm \frac{2}{\sqrt{3}}$ (b) $x = 1 \pm \frac{1}{\sqrt{3}}$ (c) $x = 2 \pm \frac{1}{\sqrt{3}}$ (d) $x = \pm 1$

41. The length of the normal at point 't' of the curve $x = a(t + \sin t), y = a(1 - \cos t)$ is **[Rajasthan PET 2001]**

(a) $a \sin t$ (b) $2a \sin^3\left(\frac{t}{2}\right)\sec\left(\frac{t}{2}\right)$ (c) $2a \sin\left(\frac{t}{2}\right)\tan\left(\frac{t}{2}\right)$ (d) $2a \sin\left(\frac{t}{2}\right)$

42. The length of normal to the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ at the point $\theta = \frac{\pi}{2}$ is **[Rajasthan PET 1999; AIEEE 2004]**

(a) $2a$ (b) $\frac{a}{2}$ (c) $\sqrt{2}a$ (d) $\frac{a}{\sqrt{2}}$

43. The area of the triangle formed by the coordinate axes and a tangent to the curve $xy = a^2$ at the point (x_1, y_1) on it is **[DCE 2001]**

(a) $\frac{a^2 x_1}{y_1}$ (b) $\frac{a^2 y_1}{x_1}$ (c) $2a^2$ (d) $4a^2$

44. The normal of the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any θ is such that **[DCE 2000]**

(a) It makes a constant angle with x-axis (b) It passes through the origin

(c) It is at a constant distance from the origin (d) None of these

45. An equation of the tangent to the curve $y = x^4$ from the point (2, 0)not on the curve is

(a) $y = 0$ (b) $x = 0$ (c) $x + y = 0$ (d) None of these

46. For the curve $by^3 (x + a)^3$ the square of subtangent is proportional to

(a) (Subnormal)$^{1/2}$ (b) Subnormal (c) (Subnormal)$^{3/2}$ (d) None of these

47. The tangent to the curve $y = ax^2 + bx$ at $(2, -8)$ is parallel to x-axis. Then **[AMU 1999]**

(a) $a = 2, b = -2$ (b) $a = 2, b = -4$ (c) $a = 2, b = -8$ (d) $a = 4, b = -4$

48. If the area of the triangle include between the axes and any tangent to the curve $x^n y = a^n$ is constant, then n is equal to

(a) 1 (b) 2 (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

49. All points on the curve $y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ at which the tangents are parallel to the axis of x, lie on a

(a) Circle (b) Parabola (c) Line (d) None of these

50. If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ cut each other orthogonally, then

(a) $a^2 + b^2 = l^2 + m^2$ (b) $a^2 - b^2 = l^2 - m^2$ (c) $a^2 - b^2 = l^2 + m^2$ (d) $a^2 + b^2 = l^2 - m^2$

51. The length of the normal at any point on the catenary $y = c\cos h\left(\frac{x}{c}\right)$ varies as

(a) (abscissa)^2 (b) (Ordinate)^2 (c) abscissa (d) ordinate

52. The point P of the curve $y^2 = 2x^3$ such that the tangent at P is perpendicular to the line $4x - 3y + 2 = 0$ is given by

(a) (2, 4) (b) (1, $\sqrt{2}$) (c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{8}, -\frac{1}{16}\right)$

53. The length of the normal to the curve $y = a\left(\frac{e^{-x/a} + e^{x/a}}{2}\right)$ at any point varies as the

(a) Abscissa of the point (b) Ordinate of the point

(c) Square of the abscissa of the point (d) Square of the ordinate of the point

54. If the parametric equation of a curve given by $x = e^t \cos t, y = e^t \sin t,$ then the tangent to the curve at the point $t = \frac{\pi}{4}$ makes with axes of x the angle

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

55. For the parabola $y^2 = 4ax$, the ratio of the subtangent to the abscissa is

(a) 1 : 1 (b) 2 : 1 (c) $x : y$ (d) $x^2 : y$

56. Tangents are drawn from the origin to the curve $y = \cos x$. Their points of contact lie on

(a) $x^2y^2 = y^2 - x^2$ (b) $x^2y^2 = x^2 + y^2$ (c) $x^2y^2 = x^2 - y^2$ (d) None of these

57. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3) then

(a) $p = 2, q = -7$ (b) $p = -2, q = 7$ (c) $p = -2, q = -7$ (d) $p = 2, q = 7$

58. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point

(a) (1, 1) (b) At no point (c) (0, 1) (d) (1, 0)

59. If the tangent and normal at any point P of parabola meet the axes at T and G respectively then

(a) ST = SG.SP (b) ST = SG = SP (c) ST $\neq$ SG = SP (d) ST = SG $\neq$ SP

60. Slope of the tangent to the curve $y = |x^3|$ at origin is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) 0

61. The line $\left(\frac{x}{a}\right)+\left(\frac{y}{b}\right)=2$, touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at point (a, b) then $n=$ **[Rajasthan PET 1998]**

(a) 1 (b) 2 (c) 3 (d) For non-zero values of n

62. The sum of the squares of intercepts made by a tangent to the curve $x^{2/3}+y^{2/3}=a^{2/3}$ with coordinate axes is **[Rajastha**

(a) a (b) $2a$ (c) a^2 (d) $2a^2$

63. The point of the curve $y = x^2 - 3x + 2$ at which the tangent is perpendicular to the $y = x$ will be

(a) (0, 2) (b) (1, 0) (c) (–1, 6) (d) (2, –2)

64. The equation of normal to the curve $\frac{x^2}{16}-\frac{y^2}{9}=1$ at the point $(8, 3\sqrt{3})$ is **[MP PET 1996]**

(a) $\sqrt{3}x + 2y = 25$ (b) $x + y = 25$ (c) $y + 2x = 25$ (d) $2x + \sqrt{3}y = 25$

65. The angle of intersection between the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ is **[Rajasthan PET 1998]**

(a) 0^o (b) 30^o (c) 45^o (d) 90^o

66. The subtangent to the curve $x^m y^n = a^{m+n}$ at any point is proportional to **[Rajasthan PET 1998]**

(a) Ordinate (b) Abscissa (c) (Ordinate)^n (d) (Abscissa)^n

67. If tangents drawn on the curve $x = at^2, y = 2at$ is perpendicular to x-axis then its point of contact is

(a) (a, a) (b) $(a, 0)$ (c) $(0, a)$ (d) (0, 0)

68. Tangents are drawn to the curve $y = x^2 - 3x + 2$ at the points where it meets x-axis. Equations of these tangents are

[Rajasthan PET 1993]

(a) $x - y + 2 = 0, x - y - 1 = 0$ (b) $x + y - 1 = 0, x - y = 2$ (c) $x - y - 1 = 0, x - y = 0$ (d) $x - y = 0, x + y = 0$

69. If the tangents at any point on the curve $x^4 + y^4 = a^4$ cuts off intercept p and q on the axes, the value of $p^{-4/3} + q^{-4/3}$ is

(a) $a^{-4/3}$ (b) $a^{-1/2}$ (c) $a^{1/2}$ (d) None of these

70. At any point (x_1, y_1) of the curve $y = ce^{x/a}$

(a) Subtangent is constant

(b) Subnormal is proportional to the square of the ordinate of the point

(c) Tangent cuts x-axis at $(x_1 - a)$ distance from the origin

(d) All the above

71. The equation of the tangent to the curve $y = 1 - e^{x/2}$ at the point where it meets y-axis is

(a) $x + 2y = 2$ (b) $2x + y = 0$ (c) $x - y = 2$ (d) None of these

72. The coordinates of the points on the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$, where tangent is inclined an angle $\frac{\pi}{4}$ to the x-axis are

(a) (a, a) (b) $\left(a\left(\frac{\pi}{2} - 1\right), a\right)$ (c) $\left(a\left(\frac{\pi}{2} + 1\right), a\right)$ (d) $\left(a, a\left(\frac{\pi}{2} + 1\right)\right)$

73. If equation of normal at a point $(m^2 - m^3)$ on the curve $x^3 - y^2 = 0$ is $y = 3mx - 4m^3$, then m^2 equals

(a) 0 (b) 1 (c) $-\frac{2}{9}$ (d) $\frac{2}{9}$

74. For a curve $\frac{(\text{Length of normla})^2}{(\text{Length of tangent})^2}$ is equal to

(a) (Subnormal)/(Subtangent) (b) (Subtangent)/(Subnormal) (c) (Subtangent/Subnormal)^2 (d) Constant

75. If the curve $y = x^2 + bx + c$, touches the line $y = x$ at the point (1, 1), the values of b and c are

(a) – 1, 2 (b) –1, 1 (c) 2, 1 (d) –2, 1

76. Let C be the curve $y^3 - 3xy + 2 = 0$. If H and V be the set of points on the curve C where tangent to the curve is horizontal and vertical respectively, then

(a) $H = \{(1,1)\}, V = \phi$ (b) $H = \phi, V = \{(1,1)\}$ (c) $H = \{(0,0)\}, V = \{(1,1)\}$ (d) None of these

77. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then

(a) $a, b \in R$ (b) $a > 0, b > 0$ (c) $a < 0, b > 0$ or $a > 0, b < 0$ (d) $a < 0, b < 0$

78. If the tangent to the curve $f(x) = x^2$ at any pint $(c, f(c))$ is parallel to line joining the points $(a, f(a))$ and $(b, f(b))$ on the curve, then a, c, b are in

(a) H.P. (b) G.P. (c) A.P. (d) A.P. and G.P. both

79. The area of triangle formed by tangent to the hyperbola $2xy = a^2$ and coordinates axes is

(a) a^2 (b) $2a^2$ (c) $\frac{a^2}{2}$ (d) $\frac{3a^2}{2}$

80. The angle of intersection between the curves $r = a\sin(\theta - \alpha)$ and $r = b\cos(\theta - \beta)$ is

(a) $\alpha - \beta$ (b) $\alpha + \beta$ (c) $\frac{\pi}{2} + \alpha + \beta$ (d) $\frac{\pi}{2} + \alpha - \beta$

81. The distance between the origin and the normal to the cure $y = e^{2x} + x^2$ at the point $x = 0$ is

(a) $2\sqrt{5}$ (b) $\frac{2}{\sqrt{5}}$ (c) $\sqrt{5}$ (d) None of these

82. If the curve $y = ax^2 - 6x + b$ passes through (0, 2) and has its tangent parallel to x-axis at $x = \frac{3}{2}$, then the value of a and b are

[SCRA 1999]

(a) 2, 2 (b) –2, –2 (c) –2, 2 (d) 2, –2

83. If at any point S of the curve $by^2 = (x+a)^3$ the relation between subnormal SN and subtangent ST be $p(SN) = q(ST)^2$ then p/q is equal to
[Rajasthan PET 1999; EAMCET 1991]

(a) $\frac{8b}{27}$ (b) $\frac{8a}{27}$ (c) $\frac{b}{a}$ (d) None of these

84. The points on the curve $9y^2 = x^3$ where the normal to the curve cuts equal intercepts from the axes are

(a) (4, 8/3), (4, −8/3) (b) (1, 1/3) (1, −1/3) (c) (0, 0) (d) None of these

85. The equation of the normal to the curve $y^2 = x^3$ at the point whose abscissa is 8, will be

(a) $x \pm \sqrt{2}y = 104$ (b) $x \pm 3\sqrt{2}y = 104$ (c) $3\sqrt{2}x \pm y = 104$ (d) None of these

86. At any point (except vertex) of the parabola $y^2 - 4ax$ subtangent, ordinate and subnormal are in

(a) AP (b) GP (c) HP (d) None of these

87. At what point the slope of the tangent to the curve $x^2 + y^2 - 2x - 3 = 0$ is zero **[Rajasthan PET 1989, 1995]**

(a) (3 0); (−1, 0) (b) $(3,0); (1,2)$ (c) (−1, 0); (1, 2) (d) (1, 2); (1, −2)

88. Let the equation of a curve be $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$. If θ changes at a constant rate k then the rate of change of the slope of the tangent to the curve at $\theta = \frac{\pi}{3}$ is

(a) $\frac{2k}{\sqrt{3}}$ (b) $\frac{k}{\sqrt{3}}$ (c) k (d) None of these

89. The equation of a curve is $y = f(x)$. The tangents at $(1, f(1)), (2, f(2))$ and $(3, f(3))$ makes angles $\frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{4}$ respectively with the positive direction of the x-axis. Then the value of $\int_2^3 f'(x)f''(x)dx + \int_1^3 f''(x)dx$ is equal to

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) 0 (d) None of these

90. $P(2,2)$ and $Q\left(\frac{1}{2}, -1\right)$ are two points on the parabolas $y^2 = 2x$. the coordinates of the point R on the parabola, where the tangent to the curve is parallel to the chord PQ, is

(a) $\left(\frac{5}{4}, \sqrt{\frac{5}{2}}\right)$ (b) (2, − 1) (c) $\left(\frac{1}{8}, \frac{1}{2}\right)$ (d) None of these

91. The number of tangents to the curve $x^{3/2} + y^{3/2} = a^{3/2}$,. where the tangents are equally inclined to the axes, is

(a) 2 (b) 1 (c) 0 (d) 4

92. If at each point of the curve $y = x^3 - ax^2 + x + 1$ the tangent is inclined at an acute angle with the positive direction of the x-axis then

(a) $a > 0$ (b) $a \le \sqrt{3}$ (c) $-\sqrt{3} \le a \le \sqrt{3}$ (d) None of these

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Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	b	a	c	a	c	a	b	a	d	b	a	c	b	b	b	a	d	d	a
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	d	d	a	d	a	c	c	b	c	b	d	a	a	b	a	d	c	d	a
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	c	c	a	b	c	a	b	c	b	d	d	d	b	c	a	d	b	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d	c	b	d	a	b	d	b	a	d	d	c	d	a	b	b	c	c	a	d
81	82	83	84	85	86	87	88	89	90	91	92								
b	a	a	a	b	b	d	d	a	c	b	c								